Thiruvalluvar University, Vellore - 632115
Name of the Programme: M.Sc. Mathematics Semester: I
Name of the Course: Real Analysis I Credits: 5
Paper type: Core Hours of teaching: 90hrs
Course Objectives
The objectives of the course is to

- work comfortably with functions of bounded variation
- study the Riemann - StieltjesIntegration
- study the convergence of infinite series, infinite product and uniform convergence and its interplay between various limiting operations.

UNIT-1: Functions of Bounded Variation 18 hours
Introduction - Properties of monotonic functions - Functions of bounded variation – Total variation - Additive property of total variation - Total variation on [a, x] as a function of x - Functions of bounded variation expressed as the difference of two increasing functions - Continuous functions of bounded variation. (Chapter - 6 : Sections 6.1 to 6.8)
UNIT-2: The Riemann - Stieltjes Integral 18 hours
Introduction - Notation - The definition of the Riemann - Stieltjes integral – Linear Properties - Integration by parts- Change of variable in a Riemann - Stieltjes integral - Reduction to a Riemann Integral - Euler’s summation formula - Monotonically increasing integrators, Upper and lower integrals - Additive and linearity properties of upper and lower integrals - Riemann's condition. (Chapter - 7 : Sections 7.1 to 7.13)
UNIT-3: The Riemann-Stieltjes Integral 18 hours
Integrators of bounded variation- Sufficient conditions for the existence of Riemann Stieltjes integrals-Necessary conditions for the existence of Riemann-Stieltjes integrals Mean value theorems for Riemann - Stieltjes integrals - The integrals as a function of the interval - Second fundamental theorem of integral calculus-Change of variable in a Riemann integral- Second Mean Value Theorem for Riemann integral-Riemann-Stieltjes integrals depending on a parameter-Differentiation under the integral sign. (Chapter - 7: 7.15 to 7.24)
UNIT-4: Infinite Series and Infinite Products 18 hours
Absolute and conditional convergence - Dirichlet's test and Abel's test – Rearrangement of series - Riemann's theorem on conditionally convergent series. Double sequences - Double series - Rearrangement theorem for double series - A sufficient condition for equality of iterated series - Multiplication of series - Cesarosummability – Infinite products.
(Chapter 8: Sections 8.8, 8.15, 8.17, 8.18, 8.20, 8.21 to 8.26)

UNIT-5: Sequence of Functions 18 hours
Pointwise convergence of sequences of functions - Examples of sequences of real - valued functions - Definition of uniform convergence - Uniform convergence and continuity - The

Cauchy condition for uniform convergence - Uniform convergence of infinite series of functions - Uniform convergence and Riemann - Stieltjes integration - Uniform convergence and differentiation - Sufficient condition for uniform convergence of a series - Mean convergence.(Chapter - 9 Sec 9.1 to 9.6, 9.8, 9.10,9.11, 9.13)

Prescribed Book

Tom M. Apostol : Mathematical Analysis, 2nd Edition, Addison-Wesley Publishing Company Inc. New York, (1997).

Reference Books

1. R. G. Bartle, Real Analysis, (1976), John Wiley and sons Inc.

2. W. Rudin, Principle of Mathematical Analysis (1976), McGraw Hill Company, New York.

3. S. C. Malik and SavitaArora, Mathematical Analysis (1991), Wiley Eastern Limited. New Delhi.

4. Sanjay Arora and BansiLal, Introduction to Real Analysis (1991), SatyaPrakashan, New Delhi.

5. A.L. Gupta and N. R. Gupta, Principle of Real Analysis (2003),Pearson Education.

E-Materials

https://ocw.mit.edu/courses/mathematics/18-100a-introduction-to-analysis-fall-2012/

Course Learning Outcomes

After the successful completion of this course, the students will be able to:

- understand the concept of functions of bounded variation.
- Discuss the Riemann integration and to solve its related problems.
- Analyse the sequences and series of function and their limits
- Acquire the knowledge of Infinite Series and Infinite products
- have knowledge of uniform convergence of sequence and series